BACCALAURÉAT GÉNÉRAL ET TECHNOLOGIQUE
ÉPREUVE SPÉCIFIQUE DES SECTIONS EUROPÉENNES
MATHÉMATIQUES – ANGLAIS

SUJET 1-A

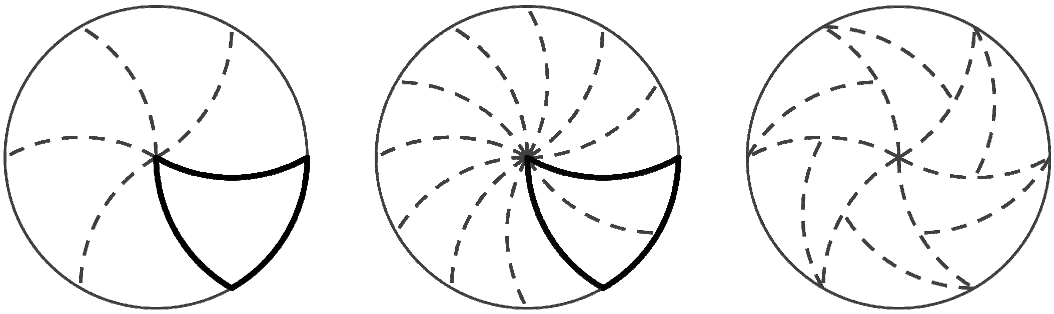
Thème : Geometry

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

How to perfectly slice pizza, according to science.

Dividing up hot pizza equally usually means a frustrating wait, inevitable failure and someone making do with a disappointing small slice. If only science had the answer, thought two mathematicians at the University of Liverpool. So they set out to find it. Joel Haddley and Stephen Worsley's new paper on the topic builds on existing circular geometry research, which recommends monohedrally tiling your pizza.

Simply put, previous mathematicians had advised carving three wavy lines through the centre of your pizza, to make six pieces like shields[1]. These pieces can then be equally subdivided into 12, and so on.



Quoting their article : "Highlighted in this figure is the shield. The shield has a line of symmetry, so we can flip each pair of tiles within each of the 6 shields."
One advantage of the new technique is that crust[2] is divided unequally, meaning crust haters can choose slices from the centre, while crust lovers can pick slices from the rim[3].

Sources: How to perfectly slice pizza, according to science | The Independent | 01/09/2016 | C. Beale
Infinite families of monohedral disk tilings | arXiv | 12/07/ 2015 | *J. Haddley, S. Worsley* | University of Liverpool

Questions :

1. Start by reading the last sentence of the text.
2. Explain what the text deals with and comment on it.

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[1] the shield = le bouclier
[2] the crust = la croûte
[3] the rim = le bord

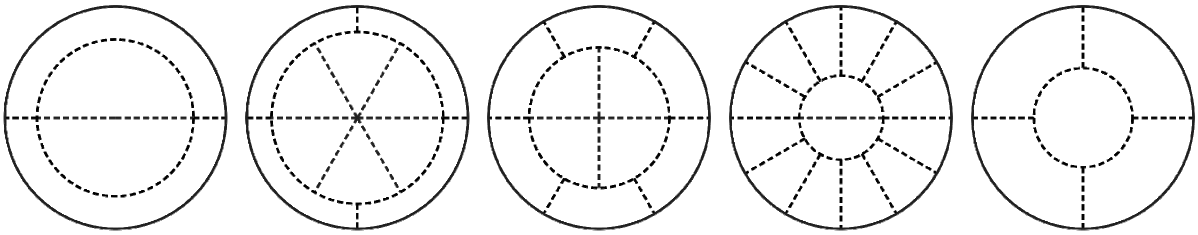
Exercise:

We want to study another way of dividing a pizza into *equal parts* between a certain number of people, some loving the crust, and some hating it. We consider:

- *c*: the number of **c**rust lovers.
- *n*: the number of people that want **n**o crust at all.

Below you have several drawings depending on the values of *c* and *n.*

Notice that in each case the parting is fair so each slice is the same area.



c=2 ; n=2 c=4 ; n=6 c=? ; n=? c=12 ; n=2 c=4 ; n=1

1. Give the values of *c* and *n* on the third picture above.

2. Draw approximately how you would cut a pizza in the same way for c=n=1.
We consider in the following questions that the pizza has a radius of 1*.*

3. On your previous drawing (for c=1 and n=1), is the radius of the inner circle equal to one half? If not, what is it equal to?

4. On the second picture where c=4 and n=6, prove that the radius of the inner circle is equal to the square root of 0.6

5. Can you find a general formula for the radius of the inner circle for *c* crust lovers and *n* crust haters?

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MATHÉMATIQUES – ANGLAIS

SUJET 2-A

Thème : Fonctions

L’usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

John Forbes Nash was born on June 13, 1928. He was a professor at Princeton University and received the 1994 Nobel Memorial Prize in Economic Sciences for his work in game theory and particularly the ideas behind the Nash Equilibrium.

The Nash equilibrium is widely used in economics as the main alternative to competitive equilibrium. It is used whenever there is a strategic element to the behaviour of agents and the "price taking" assumption of competitive equilibrium is inappropriate.

The first use of the Nash equilibrium was in the Cournot duopoly as developed by Antoine Augustin Cournot in his 1838 book. Both firms produce a homogenous product: given the total amount supplied by the two firms, the (single) industry price is determined using the demand curve. Cournot assumed that each firm chooses its own output to maximize its profits given the output of the other firm. The Nash equilibrium occurs when both firms are producing the outputs which maximize their own profit given the output of the other firm.

Sources : https://en.wikipedia.org/wiki/Economic_equilibrium

https://fr.wikipedia.org/wiki/John_Forbes_Nash

1) Read aloud the last paragraph of the text (From “The first use” down to “the other firm”)

2) Explain what the text deals with and comment on it.

Exercise:

The supply function (in 10 €) of the potatoes in a market is given by:

$O(q) = 2q^2 + 1.5q + 17;$ when the demand function (in 10€) is given by:

$D(q) = q^2 - 20q + 110$. The quantity $q,\ (q < 10\,)$represents the weight of potatoes expressed in tonne.

1) What type of function are O and D ? How do you call the graph of this function?

2)

a) Find the preimage of 43 under O and interpret it in the context of the situation.

b) Find the image 5 under D and interpret it in the context of the situation

c) Find the range of the functions O and D.

3) Comment on the variations of the functions O and D.

4) The equilibrium is the point at where the demand curve and supply curve intersect. By using an equation, find the equilibrium quantity and price potatoes.

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MATHÉMATIQUES – ANGLAIS

SUJET 3-A

Thème : Sequences

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

There are many applications of sequences. To solve problems involving sequences, it is a good strategy to list the first few terms, and look for a pattern that aids in obtaining the general term. When the general term is found, then one can find any term in the sequence without writing all the preceding terms.
Sequences are useful in our daily lives as well as in higher mathematics. For example, the interest portion of monthly payments made to pay off an automobile or home loan, and the list of maximum daily temperatures in one area for a month are sequences.

1. Read the text.
2. Explain what the text deals with and comment on it.

Exercise:

John wants to buy a computer that costs 2100 euros. He decides to pay 145 euros the first month and the following instalments with a monthly increase of 3% the ten following months. On the twelfth month, he will pay what is left.

1) Explain the difference between arithmetic sequences and geometric sequences.

2) Explain why the sequence in this example is a geometric sequence. What is the common ratio?

3) Write the recursive formula of this sequence and use it to calculate what John will have to pay the second month.

4) Use the explicit formula of the sequence to calculate what John will have to pay the eleventh month.

5) What is the total amount that will be paid by John at the end of the 11^{th} month?

6) Calculate what John will have to pay the 12^{th} month.

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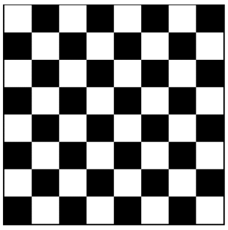
MATHÉMATIQUES – ANGLAIS

SUJET 4-A

Thème : Sequences

Ce sujet comporte 1 page

Rice and Chessboards



3000 BC.
The king Belkib of India was walking in his garden when he was attacked by some bandits. He was rescued by a mysterious knight, known as Sissa the Wise.
The king then promised the knight a great reward.
-"I will give you 1000 tons of rice"
But Sissa made another proposition:
-"Look at this chessboard," he said, "You are going to give me this chessboard. But before you do, you are going to put rice on each square of the board. You will put 1 grain of rice on the first square, 2 grains on the second square, 4 grains on the third, 8 grains on the fourth square and so on, doubling the amount for each square."
Belkib agreed immediately.

1. Read the first paragraph of the text.
2. Comment on the text.

Exercise:

Let $(u_n)_1^{64}$ be the sequence given by the number of grains on each square of the chessboard, for *n* from 1 to 64.

1. What is $u_1$equal to? What is u_4 equal to? u_{10}?
2. Is (u_n) an arithmetic sequence? A geometric sequence? Explain.
3. Can you express u_n in terms of n?
4. A grain of rice weighs 0.01g. Is it possible for Belkib to grant Sissa's offer?

Reminders:
In 2012, the total production of rice in the world was about 738 million tonnes.
(Source : http://faostat.fao.org/site/567/DesktopDefault.aspx?PageID=567#ancor)

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MATHÉMATIQUES – ANGLAIS

SUJET 5-A

Thème : Sequences

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

In ancient times, people didn't use money; they swapped or bartered[1] things, such as cattle. Let's face it though, cattle aren't that easy to transport, so an alternative was needed. In the early days of money, people used natural objects that were fairly rare and small enough to be carried easily; these could be anything from the shells of cowry sea snails[2] to feathers. Metal coins started to be used around 700 BC in Lydia, which is now a part of Turkey, and China. Even so, if people were travelling long distances, or buying or selling huge sums, they still had to carry large, heavy bags of coins. In 12th century China, this problem was solved with the invention of paper money. That's not the end of the story though. Nowadays, most of the world's money is stored on computer and is transferred electronically between banks, shops and people using cards. You might already have your own bank account, and maybe your savings are already earning "interest".

Extract from: "FROM ZERO TO INFINITY (And Beyond) Cool maths stuff you need to know" by Dr Mike Goldsmith (2011)

1. Read the text from "Metal coins..." to "...using cards."
2. Explain what the text deals with and comment on it.

Exercise:

A man deposited 19,000 £ into his new account in a bank.
The interest rate is 4.5 % per year. He withdraws 810 £ at the end of every year.

All results will be given with whole numbers.

1. What is the amount on the bank account (in pounds) after one year?
Knowing that one Euro is equal to 0.76 £, calculate this amount in Euros.

2. Let's consider the sequence $C_{n+1} = 1.045\ C_n - 810$ and $C_0 = 19{,}000$.
a) Justify that C_n is the amount (in pounds) after n years.
b) Calculate C_1, C_2 and C_3
c) Calculate the first three terms of the sequence $U_n = C_n - 18{,}000$
d) Is (U_n) an arithmetic sequence? A geometric sequence?

3. Prove that $C_n = 1000\ (1.045)^n + 18{,}000$; (we can consider (U_n) as a geometric sequence)

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[1] barter = exchange
[2] cowry sea snail = precious snail

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MATHÉMATIQUES – ANGLAIS

SUJET 6-A

Thème : Statistics

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Part of the history of statistics

The word statistics have been derived from Latin word "Status" or the Italian word "Statista", meaning of these words is "Political State" or a Government. Shakespeare used a word Statist is his drama Hamlet (1602). In the past, the statistics was used by rulers. The application of statistics was very limited but rulers and kings needed information about lands, agriculture, commerce, population of their states to assess their military potential, their wealth, taxation and other aspects of government

The arithmetic mean, although a concept known to the Greeks, was not generalised to more than two values until the 16th century. The invention of the decimal system by Simon Stevin in 1585 seems likely to have facilitated these calculations. This method was first adopted in astronomy by Tycho Brahe who was attempting to reduce the errors in his estimates of the locations of various celestial bodies.

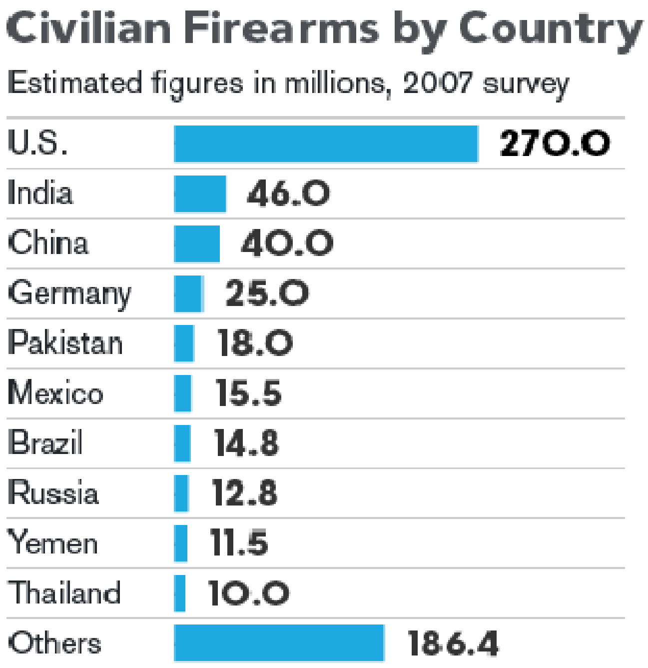
Although the original scope of statistics was limited to data useful for governance, the approach was extended to many fields of a scientific or commercial nature during the 19th century. The mathematical foundations for the subject heavily drew on the new probability theory, pioneered in the 16th century in the correspondence amongst Gerolamo Cardano, Pierre de Fermat and Blaise Pascal. Christiaan Huygens (1657) gave the earliest known scientific treatment of the subject.

Source: https://en.wikipedia.org/wiki/History_of_statistics and
http://www.emathzone.com/tutorials/basic-statistics/history-of-statistics.html

1. Start the interview by reading the first six lines.
2. Explain what the text deals with and comment on it.

Exercise:

In USA, there is a debate about the guns in this country.
Here, you can find a chart about civilian firearms by country in 2007.



1- Knowing that at this time there were 302 million of people in USA and 147 million in Russia, what is the percentage of civilian firearms in USA, in Russia?

2-

a) With your calculator, compute the mean and the median of the table chart.
b) Compute the quartiles.
c) Draw the box plot of your results.
d) What could you say about these results?

3- In 2013, there were 357 million civilian firearms in USA with a population of 317 million. What is the percentage of civilian firearms in USA at this time?

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MATHÉMATIQUES – ANGLAIS

SUJET 7-A

Thème : Astronomie et probabilités

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Mathematics and planet discovery

Mary Somerville, the 19th century mathematician and astronomer who overcame huge disadvantages to pursue her career at a time when women were discouraged from studying science, was announced as the winner of the RBS[1] poll on Wednesday.

Her face will become increasingly familiar to Scots from next year, when it will start to appear on the bank's new plastic £10 notes.

Her discussion of a hypothetical planet perturbing Uranus in the sixth edition (1842) of her book *The connection of the physical sciences* led Adams to his investigation and subsequent discovery of Neptune. Somerville wrote:

Those [*the tables*] *of Uranus, however, are already defective, probably because the discovery of that planet in* 1781 *is too recent to admit of much precision in the determination of its motions*[2]*, or that possibly it may be subject to disturbances*[3] *from some unseen planet revolving about the sun beyond the present boundaries*[4] *of our system. If, after a lapse of years, the tables formed from a combination of numerous observations should be still inadequate to represent the motions of Uranus, the discrepancies*[5] *may reveal the existence, nay*[6]*, even the mass and orbit, of a body placed forever beyond the sphere of vision.*

Sources : http://www.independent.co.uk/news/people/mary-somerville-pioneering-scientist-to-appear-on-new-rbs-10-note-a6865631.html
http://www-history.mcs.st-andrews.ac.uk/Biographies/Somerville.html

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[1] RBS : Royal Bank of Scotland
[2] motions : movements
[3] disturbances : interferences
[4] boundaries : limits
[5] discrepancies : differences
[6] nay : (here) indeed

Exercise:

This exercise is about the relations that could exist between extrasolar planets and the star they orbit. The probability of existence of planets is not the same for different kinds of stars.
During an astronomical observation to find gas giant planets among 200 star systems, 28 gas giant planets have been discovered orbiting 75 metal-rich stars, and only 2 for the 125 metal-poor stars. Each time, the gas giant planet is found alone in its star system, without any other gas giant planet.

Let G be the set of the stars which system has only one gas giant planet and M be the set of metal-rich stars. Let's choose a star at random.

1) What are the probabilities of the events, *M* , not *M* and *G* ?

2) What's the meaning of the probabilities P(G/M) and(P(G/not M)? What's their value? Comment on the results.

3) Only one gas giant planet is orbiting a star.
What is the probability for this star to be a metal-rich one? Comment on this result.

4) Compare the probabilities for the star to be metal-rich, before and after the discovery of a gas giant planet orbiting around. What are the strong and the weak points of this method to find out if a star is metal-rich or not?

Source : https://dev-lesia.obspm.fr/webjaxe/sites/main-statistique/pages_stat-mini-projet/controle.html

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MATHÉMATIQUES – ANGLAIS

SUJET 8-A

Thème : Probabilités

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The Monty Hall problem

The Monty Hall problem is a counter-intuitive probability puzzle (Monty Hall is the name of a television show host[1]):

- There are 3 doors, behind which are two goats[2] and a car.
- You pick a door (call it door A). You're hoping for the car of course.
- Monty Hall examines the other doors (B & C) and always opens one of them with a goat (both doors might have goats; he'll randomly pick one to open)

Here is the game: Do you stick with door A (original guess) or switch to the other unopened door? Does it matter?

1°) Explain your intuitive choice.
2°) a) What is your chance of winning if you stick with door A?
b) What is your chance of winning if you pick randomly one of the two unopened doors that are left, including door A?
3°) a) What other strategy can you use?
b) Assuming you had chosen a goat at the first guess, which you cannot actually know by now, what is your winning chance if you choose to switch to the other door in a second time?
c) What is your chance of having chosen a goat at the first guess?
d) Finally, deduce from b) and c) your chance of winning if you change your original guess to the other possible winning door?

[1] a host = un animateur
[2] a goat = une chèvre

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MATHÉMATIQUES – ANGLAIS

SUJET 9-A

Thème : Prime numbers: Primality test

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

A prime number (or a prime) is a *natural number* greater than 1 that has no positive divisors other than 1 and itself. A natural number greater than 1 that is not a prime number is called a *composite number*.
Eratosthenes of Cyrene was a Greek mathematician, geographer, poet, astronomer, and music theorist.

He is best known for being the first person to calculate the circumference of the Earth, which he did by applying a measuring system using stadia, which was a standard unit of measure during that time period. His calculation was remarkably accurate. He was also the first to calculate the tilt[1] of the Earth's axis (again with remarkable accuracy). Additionally, he may have accurately calculated the distance from the Earth to the Sun and invented the leap day. He created the first map of the world incorporating parallels and meridians, based on the available geographical knowledge of the era.

Eratosthenes proposed a simple algorithm for finding prime numbers. This algorithm is known in mathematics as the Sieve of Eratosthenes.

In mathematics, the sieve of Eratosthenes, one of a number of prime number sieves, is a simple, ancient algorithm for finding all prime numbers up to any given limit. It does so by iteratively marking as composite, i.e., not prime, the multiples of each prime, starting with the multiples of 2. The multiples of a given prime are generated starting from that prime, as a sequence of numbers with the same difference, equal to that prime, between consecutive numbers. This is the sieve's key distinction from using trial division to sequentially test each candidate number for divisibility by each prime.

1. Read the first paragraph of the text.
2. Comment on the text.

[1] tilt = inclinaison

3. Using the text and the grid, find all the prime numbers of this list:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

4. How many prime numbers are there before 100?

5. There was a leap day this year, and not the previous one. Can you guess what a leap day is?

6. Do you know other famous Greek mathematicians?

Source : https://en.wikipedia.org/wiki/Eratosthenes

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MATHÉMATIQUES – ANGLAIS

SUJET 11-A

Thème : Geometry and sequences

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Fractals and Koch snowflake

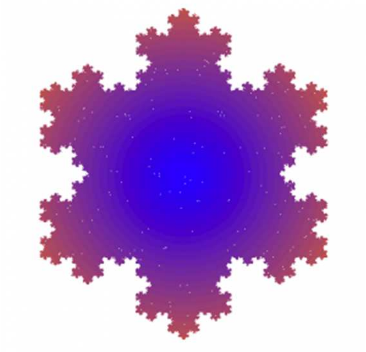
'Fractals' is a new branch of mathematics and art.

Most physical systems of nature and many human artifacts are not regular geometric shapes of the standard geometry derived from Euclid. Fractal geometry offers almost unlimited ways of describing, measuring and predicting these natural phenomena. Fractals are the best existing mathematical descriptions of many natural forms, such as coastlines, mountains or parts of living organisms. Many people are fascinated by the beautiful images termed fractals. Extending beyond the typical perception of mathematics as a body of complicated, boring formulas, fractal geometry mixes art with mathematics to demonstrate that equations are more than just a collection of numbers.

If you look carefully at a fern leaf[1], you will notice that every little leaf part of the bigger one has the same shape as the whole fern leaf. You can say that the fern leaf is self-similar. The same is with fractals: you can magnify them many times and after every step you will see the same shape, which is characteristic of that particular fractal.

From http://www.fractal.org/Bewustzijns-Besturings-Model/Fractals-Useful-Beauty.htm

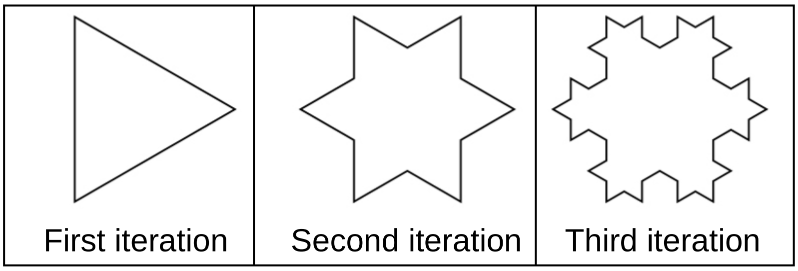
1. Read the text from the beginning down to "images termed fractals".
2. Comment on the text.



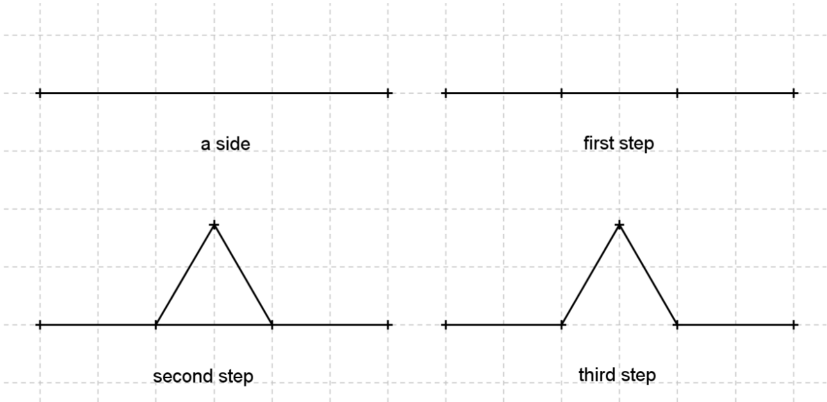
[1] fern leaf : feuille de fougère

Exercise

The Koch snowflake is a mathematical curve and one of the earliest fractal curves to have been described.



The Koch snowflake can be built by starting with an equilateral triangle.
To get the next iteration, we follow, for each side of the figure, the three steps shown below.



1. Describe each step.

2. We consider that n is the number of iteration, c_n is the number of sides of the figure at iteration n and l_n is the length of this side.
Complete the following table:

Iteration, n	1	2	3
Number of sides, c_n			
Length of a side, l_n	1		

3. **a.** What type of sequence are c_n and l_n ?
b. Deduce the expression of c_n and l_n in terms of n.

4. **a.** Give a way to calculate the perimeter P_n of the Koch snowflake at the n^{th} iteration.
b. Deduce that $P_n = 3 \times \left(\frac{4}{3}\right)^{n-1}$.
c. Deduce the limit of the perimeter of the Koch snowflake as the number of iterations tends to infinity.

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MATHÉMATIQUES – ANGLAIS

SUJET 12-A

Thème : Numbers

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π is wrong !

I know it will be called blasphemy by some, but I believe that π is wrong. For centuries π has received unlimited praise; mathematicians have waxed rhapsodic[1] about its mysteries, used it as a symbol for mathematics societies and mathematics in general, and built it into calculators and programming languages. (…) I am not questioning its irrationality, transcendence, or numerical calculation, but the choice of the number on which we bestow[2] a symbol conveying deep geometric significance. The proper value, which does deserve all of the reverence and adulation bestowed upon the current impostor, is the number now unfortunately known as 2π.

I do not necessarily feel that π can or even should be changed or replaced with an alternative, but it is worthwhile to recognize the repercussions of the error as a warning and a lesson in choosing good notational conventions to communicate mathematical ideas. (…)

The most significant consequence of the misdefinition of π is for early geometry and trigonometry students who are told by mathematicians that radian measure is more natural than degree measure. In a sense it is, since a quarter of a circle is more naturally measured by 1.57… than by 90. Unfortunately, this beautiful idea is sabotaged by the fact that π isn't 6.28…, which would make a *quarter* of a circle or a quadrant equal to a *quarter* of π radians ; a *third* of a circle, a *third* of π radians, and so on… (…). An enlightening analogy would be to leave clocks the way they are but define an hour to be 30 minutes. In that case, 15 minutes or a quarter of a clock would indeed be called half an hour, *just as* a quarter of a circle is half of π in mathematics!

From Bob Palais, *The Mathematical Intelligencer*, 2001

1. Read the article from "*I know it will be called blasphemy*…" to "*deep geometric significance*". Explain what the text deals with and comment on it.
2. Explain what the text deals with and comment on it.

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[1] to wax rhapsodic = to praise something excessively

[2] to bestow = to confer

Exercise:

1. a. What is a rational number?
   b. Can you give a few examples?
2. a. What is an irrational number?
   b. Can you give a few examples?
3. a. Compute $\frac{4}{5}+\frac{13}{15}$; express the result as a mixed number.
   b. Prove that the sum of two rational numbers is a rational number.
4. "The sum of an irrational number and a rational number is always irrational".
   What do you think of this statement?
5. What about the sum of two irrational numbers?

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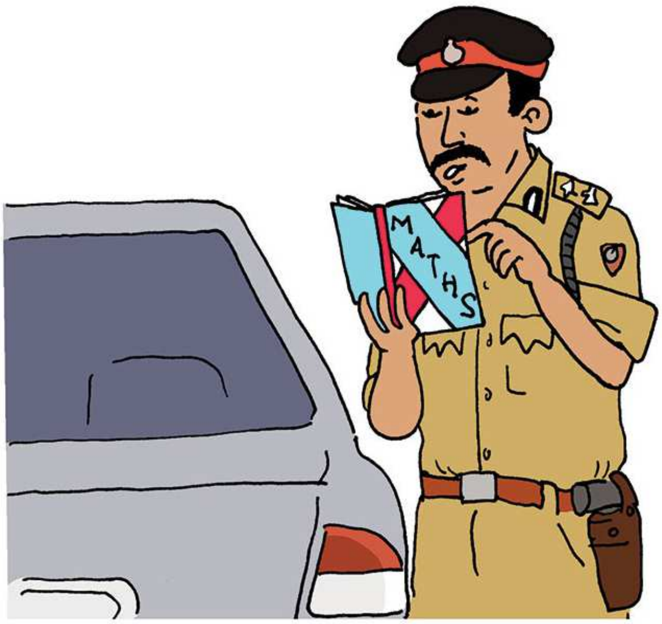
MATHÉMATIQUES – ANGLAIS

SUJET 13-A

Odd arguments – Reasoning about parity

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

An odd argument



In the first weeks of January, authorities in the Indian city of Delhi ran a traffic-calming experiment whereby cars with licence plates ending in an even number could only drive on even-numbered days, and vice versa. The rules are being enforced by thousands of volunteer officers following "Gandhigiri" methods – issuing polite reprimands and gifts of flowers, rather than fines.

Things got off to a bumpy start when, on 1 January, volunteer traffic wardens approached a man driving a car with a plate ending in a zero. The man argued that zero was neither an odd nor an even number, and he should be permitted to continue his journey. The driver said he was a student of mathematics and commerce, and insisted that "zero did not fit into either category".

Eventually the guards admitted defeat, offered him a rose – which he refused – and moved on.

From NewScientist (23 January 2016)

Nb: Gandhigiri is a word used to express the idea of non-violence (in India).

1. Read the second paragraph of the text out loud.
2. Explain what the text deals with and comment on it.

Exercise

a. What are the definitions of even numbers and odd numbers?
b. Is 0 an even number or an odd number?
c. Do you know an easy way to discriminate between even numbers and odd numbers?
d. Prove that the sum of two odd numbers is an even number.
   What about the sum of three odd numbers?
e. Is the square of an odd number always an odd number? Prove it.
f. Consider the sum of the first 10 numbers: 1+2+3+4+5+6+7+8+9+10.
   Can you change some of the plus signs into minus signs so that the resulting sum is 0?
   For example, 1 + 2 – 3 – 4 – 5 + 6 - 7 - 8 + 9 + 10 = 1.
   This is close to 0, but not exactly 0.

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MATHÉMATIQUES – ANGLAIS

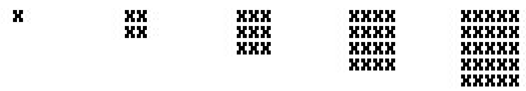
SUJET 15-A

Thème : Sequences

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

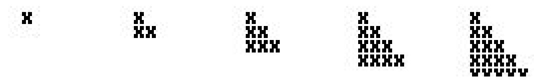
Square and triangular numbers

Everyone knows how to compute a square of a given number especially if it's not too large. $2^2=4$, $3^2=9$, $4^2=16$ and so on. Why is this operation of multiplying a number by itself called squaring? The reason is best understood from the following picture.



A square with a side length of n can be viewed as comprising a grid of nxn smaller squares of size 1x1.

Triangular numbers owe their name to a similar construction, now of triangles. Triangular numbers are in the form 1, 3, 6, 10, 15 … For every positive integer n, the number T_n is triangular and describes the number of points in a triangular shape with n points on a side as shown below.



n = 1 n = 2 n = 3 n = 4 n = 5

In fact, in a group of n people, the number of distinct wine glass clinks[1] after a toast is the triangular number T_{n-1}.

Source: http://www.cut-the-knot.org/do_you_know/numbers.shtml

1. Read the paragraph about triangular numbers (lines 7 to 10).
2. Explain what the text deals with and comment on the text.

[1] Clink = the sound made when two glasses touch

Exercise:

1. Using the method of your choice, find a recursive formula between the n^{th} term and $(n+1)^{th}$ term.
2. The number 2016 is a triangular number. Using the general formula: $T_n = \left(\frac{n(n+1)}{2}\right)$, find *n* such that $T_n = 2016$.
3. In a group of 20 people, each person shakes the hand of everyone. How many distinct handshakes are there?

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MATHÉMATIQUES – ANGLAIS

SUJET 16-A

Thème : Conditional probabilities

L'usage de la calculatrice est autorisé. Ce sujet comporte 1 page.

To determine life expectancy, we believe that it varies with information about health or habits such as tobacco or alcohol. Life expectancy changes whenever we add further relevant information to what it called *reference class* – that is, the class of people who are like the subject in the relevant respects for which the statistics are known. Because the probabilities that determine life expectancy are based on a reference class, they are *conditional probabilities.*

Adapted from William J. Talbott *Human Rights and Human Well-Being,* *July 2010*

1) Read the text except the last sentence.
2) Comment on this text.

Exercise

Part A

The Census Bureau has estimated the following survival probabilities for men:

1. probability that a man lives at least 70 years: 80%;
2. probability that a man lives at least 80 years: 50%.

What is the probability that a man lives at least 80 years given that he has just celebrated his 70th birthday?

Part B

In 2016, every public place will be equipped with defibrillators. Moreover, a survey shows that about 71% of the population wishes to be taught how to accomplish healthcare routine. If this rate is reached,

- The probability that a heart failure occurs in front of a witness who knows how to accomplish healthcare routine would be 0.5.
- The probability of survival if a witness who knows how to accomplish healthcare routine acts would be 0.25 or 0.046 else.

In France, among the 55,000 people who are victim of cardiovascular accident, only 1,650 survive.

How many more lives could be saved if those conditions were filled?
Explain what to do to find the answer.

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MATHÉMATIQUES – ANGLAIS

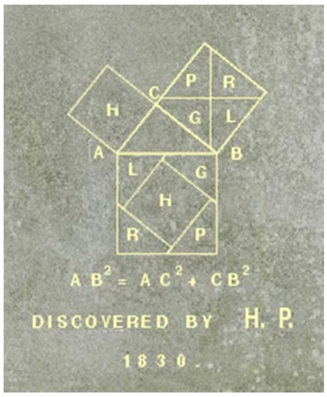
SUJET 17-A

Thème : Geometry

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

Pythagorean Theorem

Henry Perigal (1801 – 1898) as an amateur mathematician whose life spanned almost the entire nineteenth century and who is now known principally for an elegant cut-and-shift proof - what is now called a dissection proof - of Pythagoras' Theorem. He himself perhaps felt that this was his most important accomplishment - his diagram was carved, presumably at his own request, on his gravestone.



It is not easy to justify Perigal's dissection in general. But there is one case where it is easy to do so, and that is where the two smaller sides of the triangle are equal (and the ratio of the hypotenuse to side is $\sqrt{2}$). This is the case apparently pictured on Babylonian tablets dating to about 1600 B.C. (thus among the earliest examples of geometrical diagrams known, and perhaps the earliest extant examples of mathematical reasoning). It is also that discussed by Plato in the dialogue *Meno*.

From https://plus.maths.org/content/dissecting-table

1. Start the interview by reading the first five lines.
2. Explain what the text deals with and comment on it.

Questions

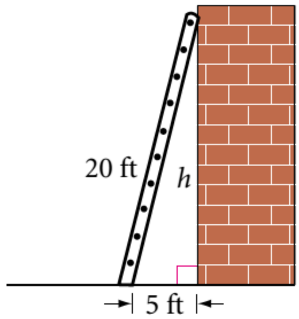
1. What does the Pythagorean Theorem assert?

2. Draw freehand the diagram described below.
   Step 1: Construct a right-angled triangle. Label the hypotenuse c and the sides a and b. Construct a square on each side of the triangle.
   Step 2: To locate the centre of the square on the longer side a or b, draw its diagonals. Label the centre O.
   Step 3: Through point O, construct line j perpendicular to the hypotenuse and line k perendicular to line j. Line k is parallel to the hypotenuse. Lines j and k divide the square on the longer side into four parts.
   Step 4: In the square on the hypotenuse, draw a line-segment starting from the midpoint of the hypotenuse parallel to the shorter side.
   Step 5: Draw three other similar line-segments starting from the other sides of the square.

3. Explain why this construction proves the Pythagorean Theorem. We don't need any proof but only visual arguments.

4. An example of using the Pythagorean theorem

   How high up on the wall will a 20-foot ladder touch if the foot of the ladder is placed 5 feet from the wall?



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MATHÉMATIQUES – ANGLAIS

SUJET 18-A

Thème : Arithmetic and probabilities

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

Magic and mathematics

Grumpelina, the Great Whodunni's beautiful assistant, placed a blindfold[1] over the eyes of the famous stage magician. A member of the audience then rolled three dice.
"Multiply the number on the first dice by 2 and add 5," said Whodunni. "Then multiply the result by 5 and add the number on the second dice. Finally, multiply the result by 10 and add the number on the third dice."
As he spoke, Grumpelina chalked up[2] the sums on a blackboard which was turned to face the audience so that Whodunni could not have seen it, even if the blindfold had been transparent.
"What do you get?" Whodunni asked.
"Seven hundred and sixty-three," said Grumpelina.
Whodunni made strange passes in the air. "Then the dice were 5, 1 and 3."

Is it magic or is it simply mathematics...? Of course it's mathematics!
If you want to understand how this illusion works, you will need a few more calculations.
If you don't, you can still believe it's magic...

Extract adapted from "Professor Stewart's hoard of mathematical treasures"
by Ian Stewart (2009)

1. Start the interview by reading the first five lines of the text ending with "third dice".
2. Explain what the text deals with and comment on it.

[1] blindfold : bandeau
[2] to chalk up : écrire à la craie

Exercise:

Let's try another "magic trick" with this time four fair 8-sided dice.
You multiply by 20 the number on you first die, and you add 15 and twice the number on your second die. Then, you multiply your result by 5, and you add the sum of twenty-six and the number on your third die. Finally, multiply your last result by 10 and add the number on the fourth die.

1. **a.** Assuming that the numbers you get on your dice are (in order): 2; 7; 5 and 6. What is the number you will get at the end of your calculations?

   b. Assuming that the numbers you get on your dice are (in order): 1; 8; 4 and 3. What is the number you will get at the end of your calculations?

2. **a.** At the end of your calculations, the answer is 9,274. Using the two previous examples, can you guess what the numbers on your dice were?

   b. Can you find a general formula that proves that your method will always work?

3. What is the probability to get the answer 7,536?

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SUJET 19-A

Thème : Probability

L'usage de la calculatrice est autorisé. Ce sujet comporte 2 pages.

The normal curve is prescribed by a specific mathematical formula which creates a bell-shaped curve; a curve with one hump and which tails away on either side.

The significance of the normal curve lies less in nature and more in theory, and in this it has a long pedigree.

In 1733, Abraham De Moivre introduced it in connection with his analysis of chance, Pierre Simon Laplace published results about it.

Carl Friedrich Gauss used it in astronomy, he decided to use his telescope to produce a more accurate calculation of the diameter of the moon. To his surprise, he discovered that every time he took a measurement, his answer was slightly different. He plotted the results and found that they formed a bell shaped curve, with most results close to the central average. (We can refer to the Gaussian law of error.)

Adolphe Quetelet used the normal curve in his sociological studies in 1835, in which he measured the divergence from "the average man" by the normal curve.

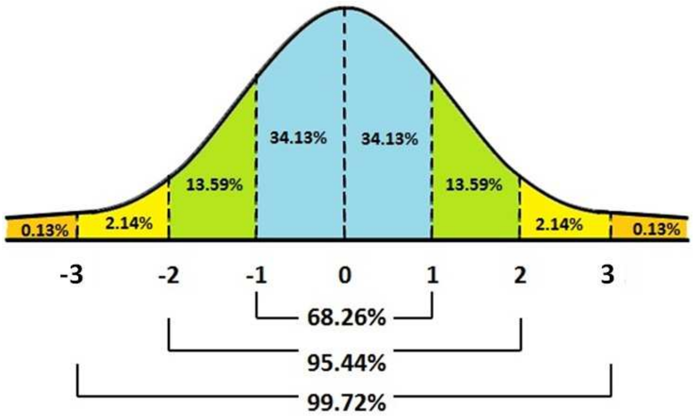
From "50 Mathematicals ideas you really need to know."
Tony Crilly.

1. Read the first lines of the text ending with "published results about it."
2. Explain what the text deals with and comment it.

<u>**Exercise**</u>

The 5 questions are independent.

If $\mu = 0$ and $\sigma = 1$ the distribution is called the **standard normal distribution**.
(μ is the mean and σ and the standard deviation)



a) 95% of students at school are between **1.1m and 1.7m** tall.
Assuming this data is **normally distributed** can you calculate the mean and standard deviation?

b) Your score in a recent test was **1 standard deviation** above the average, how many people (in %) scored **lower** than you did?

c) The mean June midday temperature in DesertTown is 36°C and the standard deviation is 3°C. Assuming this data is normally distributed, how many days in June would you expect the midday temperature to be between 39°C and 42°C ?

d) In a population that is normally distributed with mean 96 and standard deviation 25. The bottom 84 % of the values are those less than?

e) The Fresha Tea Company pack tea in bags marked as 250g. A large number of packs of tea were weighed and the mean and standard deviation were calculated as 255 g and 2.5 g respectively. Assuming this data is normally distributed, what percentage of packs are underweight?